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Abstract

Few studies have explored the source of infants' ordinal knowledge, and those that have are equivocal regarding the underlying representational system. The present study sought clear evidence that the analog number system (ANS), which underlies children's cardinal knowledge, may also support ordinal knowledge in infancy. Ten- to 12-month-old infants' were tested with large sets (>3) in an ordinal choice task in which they were asked to choose between two hidden sets of food items. The difficulty of the comparison varied as a function of the ratio between the sets. Infants reliably chose the greater quantity when the sets differed by a 2:3 ratio (4v6 and 6v9), but not when they differed by a 3:4 ratio (6v8) or a 7:8 ratio (7v8). This discrimination function is consistent with previous studies testing the precision of number and time representations in infants of roughly this same age, thus providing evidence that the ANS can support ordinal judgments in infancy. The findings are discussed in light of recent proposals that different mechanisms underlie infants' reasoning about small and large numbers.

Keywords: analog magnitudes; ordinal choice; ordinal knowledge; number; Weber fraction; infancy

Analog magnitudes support large number ordinal judgments in infancy

It was long thought that children develop an understanding of number slowly, and only after many years of experience and education. The last few decades, however, have seen a growing body of evidence that number knowledge is a core domain of cognition that supports numerical estimation, discrimination, and nonsymbolic arithmetic, and which is present at birth, and shared across a wide range of species (Feigenson, Dehaene, & Spelke, 2004; Gallistel, 1990; Mou & vanMarle, 2014; Vallortigara, Chiandetti, Rugani, Sovrano, & Regolin, 2010). Core number knowledge may also provide the foundation upon which formal mathematical knowledge may be built. For example, children's first symbolic number representations, the verbal count list (e.g., "one", "two", "three", etc.), gain their cardinal meaning (e.g., understanding that "three" refers to a set of exactly three items) in part from mappings made between each number word and its underlying preverbal magnitude representation that derive from the approximate number system (ANS) (Carey, 2004; Gallistel & Gelman, 1992, 2005; Gelman & Gallistel, 1978; Spelke, 2011; Spelke & Tsivkin, 2001; cf. LeCorre & Carey, 2007). Because cardinal knowledge in preschool is a critical predictor of mathematics achievement and risk for mathematics learning disability (MLD) (Chu, vanMarle, & Geary, 2013; vanMarle, Chu, Li, & Geary, 2014), understanding the core systems underlying this knowledge continues to be a central goal of the study of early mathematical cognition.

Because the majority of studies focus on the source of cardinal meaning, much less is known about the foundations of other important components of children's number knowledge, in particular their early understanding of *ordinality* ('more than' and 'less than' relations), which is also a critical predictor of mathematics achievement and risk for MLD (Chu et al., 2013; Lyons & Beilock, 2011). The present study therefore asks whether the same core mechanism that underlies infants' sensitivity to numerical magnitude (i.e., cardinality), can also support their ability to make ordinal judgments.

Nonverbal Representation of Number

Being able to make ordinal judgments (i.e., determining which of two sets of items is "larger" or "smaller") is a fundamental capacity that allows animals to choose, for example, the richer of two foraging patches, or decide whether they should attack or flee from an invading group of marauders (Brannon & Terrace, 2002; Gallistel, 1990; Wilson, Britton, & Franks, 2002). Such a skill would have been highly adaptive in our species' evolutionary history. But how are these judgments made?

Logically, in order to judge which of two sets is larger or smaller, one must first represent the magnitude of the sets. Over three decades of research suggest that humans share with other species a nonverbal mechanism that represents number as noisy analog magnitudes. This mechanism supports a variety of preverbal numerical abilities, including infants' ability to discriminate quantities within and across sensory modalities (Brannon, Lutz, & Cordes, 2006; Cordes & Brannon, 2008a; Feigenson, 2011; Lipton & Spelke, 2003; vanMarle & Wynn, 2006, 2009; Xu & Spelke, 2000), and to compute arithmetic operations (McCrink & Wynn, 2004, 2007). The approximate number system (ANS) is believed to be present at birth (Izard, Sann, Spelke, & Streri, 2009), and continues to contribute to numerical estimation and reasoning throughout adulthood (Barth, Kanwisher, & Spelke, 2003; Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Brannon & Merritt, 2011; Dehaene, 1997; Feigenson et al., 2004; Gallistel, 1990; Mou & vanMarle, 2014).

The hallmark of the ANS is that performance is limited by the proportionate difference between two quantities (i.e., their ratio), in accord with Weber's Law. For example, it is easier to discriminate 10 from 20, than it is to discriminate 90 from 100, even though the absolute difference (i.e., 10) is the same in both cases. Such ratio-dependent performance is seen not just in adults, but also in preverbal infants and many nonhuman animal species (Brannon, 2003; Gallistel & Gelman, 1992; Moyer & Landauer, 1967; see Gallistel, 1990 and Vallortigara et al., 2010 for reviews of animal findings). For example, at 6 months of age, human infants can discriminate sets differing by a 1:2 ratio, but not a 2:3 ratio. In both the auditory and visual modalities, infants of this age successfully discriminate 4 from 8, 8 from 16, and 16 from 32 (1:2 ratio), but not 4 from 6, 8 from 12, or 16 from 24 (2:3 ratio) (Lipton & Spelke, 2003; Xu, 2003; Xu & Spelke, 2000, Xu, Goddard, & Spelke, 2005). The fact that infants successfully discriminate 4 vs. 8, but not 8 vs. 12 or 16 vs. 24 which have equal or larger absolute differences, is strong evidence that infants' performance is limited by ratio, implicating the ANS. By 9 months, infants successfully discriminate sets differing by a 2:3 ratio, but not a 4:5 ratio (e.g., 8v10 sounds, Lipton & Spelke, 2003), suggesting that the underlying magnitude representations become more precise as infants get older, a trend which continues into adulthood (Halberda & Feigenson, 2008). The same discrimination function and developmental change in precision is also seen for duration (vanMarle & Wynn, 2006; Brannon, Suanda, & Libertus, 2007), suggesting the ANS represents both continuous and discrete quantities, as seen in nonhuman animals (Gibbon, 1977; Gibbon & Church, 1990; Meck & Church, 1983).

Emergence of Ordinal Knowledge in Infancy

Only a handful of studies have explored whether and how infants determine ordinal relations between sets. Because the ANS is believed to possess ordinal structure (i.e., the magnitudes are inherently ordered), some have suggested that it is the source of young children's understanding of ordinality in the verbal count list (Gallistel & Gelman, 1992; Gelman &

Gallistel, 1978). This raises the question of whether it may also underlie ordinal judgments in infancy; however, no studies to our knowledge have directly tested this possibility.

Brannon (2002) explored infants' sensitivity to ordinal relations using an habituation procedure and found that 11-, but not 9-month-olds, were able to detect the reversal of an ordinal sequence. For example, if habituated to sequences that increased in numerosity (e.g., 4-8-16), at test infants looked longer at sequences that decreased in numerosity (e.g., 12-6-3). In a subsequent study, Suanda, Thompson, and Brannon (2008) showed that 9-month-olds can in fact detect such reversals, but only under especially supportive conditions, when number, size, and cumulative area were all confounded. In contrast, they failed when non-numerical cues were controlled and notably, they also failed to detect a reversal in the size of a single square (e.g., small-medium-large vs. large-medium-small), suggesting that their lack of sensitivity may be general, rather than specific to numerical ordinal relationships.

Even younger infants detect reversals when given multiple cues to help them detect the relationship. Picozzi, de Hevia, Girelli, and Cassia (2010) found that when the color of the items changed on each trial, helping to demarcate the sequences, 7-month-old infants successfully detected ordinal reversals, even controlling for non-numerical cues (e.g., number, surface area, density, etc.). In the same study, infants as young as 4 months successfully detected reversals for the size of single elements in some cases (i.e., ascending but not descending sequences) suggesting that a general sensitivity to ordinal relations is available, albeit in a limited way, from very early in life (Macchi-Cassia, Picozzi, Girelli, & de Hevia, 2012).

While these and earlier studies (e.g., Cooper, 1984) shed light on when ordinal abilities emerge in infancy, they do not address what mechanism underlies these abilities. Although not always explicit in the literature, the ANS has often been assumed to be the underlying mechanism, with researchers using number sets that fall within the discrimination threshold for the age being tested (e.g., using values that differ 2-fold for infants of at least 6 months of age, Brannon, 2002; Suanda et al., 2008). This is somewhat surprising given that infants may use specialized processes – an object tracking system (OTS) – rather than the ANS for making ordinal judgments when the sets are small (i.e., <4 items – Feigenson & Carey, 2005; Feigenson, Carey, & Hauser, 2002). The OTS consists of a set of indexes that can "point" to objects in the world and keep track of them as they move and undergo occlusion (Kahneman, Treisman, & Gibbs, 1992; Leslie, Xu, Tremoulet, & Scholl, 1998; Pylyshyn & Storm, 1988; Scholl, 2001). Importantly, the OTS can track only as many objects as it has indexes, which in adults, appears to be about four (cf. Alvarez & Cavanagh, 2004; Alvarez & Franconeri, 2007).

One task that taps ordinal abilities and has been used with both infants (e.g., Feigenson et al., 2002) and nonhuman animals (e.g., Beran & Beran, 2004; vanMarle, Aw, McCrink, & Santos, 2006) is the *ordinal choice task* in which subjects are allowed to choose between two hidden sets of food items. The task measures ordinal skills by virtue of the fact that subjects are always expected to pick the larger amount, provided they can discriminate the two quantities of food. Showing a systematic preference for the larger (or smaller) amount requires that infants not only know that the two quantities differ (discrimination), but they must also be able to determine the ordinal relation ('more than' or 'less than') between the quantities. Using food is important because animals are known to be motivated to maximize food when given a choice (e.g., Beran, 2001, 2007; Boysen & Berntson, 1995; Hauser, Carey, & Hauser, 2000; vanMarle, Aw, McCrink, & Santos, 2006), motivating the prediction that they should always select the larger amount. If infants cannot quantify and compare the quantities, then they should be equally likely to choose the larger or smaller amount, such that half the infants should choose the larger and half choose the smaller.

Using this task, recent studies showed that 10- to 12-month-olds reliably chose the larger amount when comparing 1v2 crackers and 2v3, but chose randomly when either set exceeded 3, even for quantities with easily discriminable ratios (1:2 ratio - 2v4 and 3v6), and even when the ratio was extremely favorable (i.e., 1v4) (Feigenson & Carey, 2005; Feigenson et al., 2002). Infants' failure with sets larger than 3, and success with sets within the capacity limit regardless of ratio, was termed the *set size signature*, and led Feigenson and colleagues to conclude that infants' ordinal judgments depend on the capacity-limited OTS, rather than the ANS. A recent study by vanMarle (2013), however, challenged these conclusions by showing that infants can in fact make ordinal judgments with sets beyond the capacity limit, but only if both comparison quantities are beyond the limit (i.e., >3). Thus, infants successfully chose the larger amount when the comparison quantities were exclusively small (1v2), or exclusively large (4v8), but were at chance with cross-boundary comparisons (2v4 and 2v8). vanMarle concluded that both systems are capable of supporting ordinal judgments, with infants using the OTS for small sets and the ANS for large sets.

Importantly, neither studies using the ordinal choice task (Feigenson & Carey, 2005; Feigenson et al., 2002; vanMarle, 2013) nor previous looking time studies (Brannon, 2002; Brannon et al., 2008; Cassia et al., 2012; Picozzi et al., 2010) have examined the precision of infants' ordinal abilities in the large number range. Thus, while the set size signature provides clear evidence for the role of the OTS in small number ordinal judgment, no studies to date have provided clear evidence for the ANS by showing ratio-dependent performance for large number ordinal judgments. Our study thus extends previous research showing that infants are sensitive to ordinal information by examining whether the ANS may underlie this ability.

The Present Study

An abundance of studies have documented the numerical discrimination function that obtains over development. It is well established that the precision of ANS number representations increases with age, allowing older infants to make finer discriminations than younger infants (for review see Mou & vanMarle, 2014). While newborn infants require a 1:3 ratio in order to discriminate two numerosities (Izard et al., 2009), by 6 months of age, infants can discriminate sets at a 1:2 ratio (Xu & Spelke, 2000), and by 9 months they can discriminate sets at a 2:3 ratio (Lipton & Spelke, 2003). Precision continues to increase with age such that 3:4 ratios are discriminable by 3 years of age, 5:6 ratios by 6 years of age, and 9:10 ratios by adulthood (Halberda & Feigenson, 2008; Pica, Lemer, Izard, & Dehaene, 2004).

Building on previous findings showing that 10- to 12-month-old infants reliably choose the larger of two large sets differing by a 1:2 ratio (4v8 and 5v10) (vanMarle, 2013; vanMarle & Wynn, 2011), the present study was designed to examine the precision of infants' large number ordinal judgments by testing four comparisons: 4v6 and 6v9 (2:3 ratio), 6v8 (3:4 ratio), and 7v8 (7:8 ratio). Based on the developmental trajectory of increasing precision, we expected infants to succeed when the quantities differed by a 2:3 ratio and fail when they differed by a 7:8 ratio. To ensure infants' performance was truly ratio-dependent, we tested infants with two different number pairs (4v6 and 6v9) at the 2:3 ratio. Given the dearth of research between 9 months and 3 years of age, it is not clear at what age children first become able to discriminate sets at a 3:4 ratio, and thus, we made no prediction about performance in the 6v8 condition.

Method

Participants

Sixty-four healthy 10- to 12-month-old infants (Age range: 9m15d to 12m17d, *M* = 11m4d) participated in the study, 16 in each condition (8 male). An additional 35 infants were tested but excluded from the final sample due to experimenter error (7 infants), fussiness/inattention (3), or failure to choose (25). Following previous studies (see Feigenson et al., 2002; vanMarle, 2013; vanMarle & Wynn, 2011), infants were considered to have failed to choose if they did not make a choice within 20 seconds. The number of children who failed to choose did not differ across conditions (two infants in 4v6, eight in 6v9, eight in 6v8, and seven in 7v8; $\chi^2(3) = 3.96$, $p = .266$) or as a function of ratio (8, 8, and 7 infants in 2:3, 3:4, and 7:8 ratios, respectively; $\chi^2(2) = .56$, $p = .755$). All participants were recruited from the greater Columbia, MO area. Infants received a small gift or $15 travel compensation for their participation. All parents consented to their child's participation.

Design

Infants were randomly assigned to one of four conditions, 4v6, 6v9, 6v8, or 7v8, with difficulty increasing as the ratio between the quantities approached 1.0 (2:3, 3:4, and 7:8, respectively). Infants first completed a short warm-up trial; a single test trial followed in which infants were asked to choose between two hidden quantities of Cheerios (a popular breakfast cereal). The order in which the amounts were hidden (small-first or large-first), and the side on which the larger amount was hidden (left or right), was counterbalanced across infants. This was accomplished by randomly assigning infants to one of four presentation orders: small-first-left (*sm-L*), small-first-right (*sm-R*), large-first-left (*lg-L*), or large-first-right (*lg-R*)[1].

Stimuli/Materials

The stimuli and materials were identical to those used in vanMarle (2013). They included a green, plastic bucket (16 cm high x 16 cm in diameter) and a small toy duck that squeaked (9 cm wide x 8 cm high), both used in the warm-up trial, and two identical opaque red plastic cups (16 cm high and 8.5 cm in diameter) into which the experimenter hid Cheerios (each O was approx. 1 cm in diameter) during the test trial. The quantities consisted of 4, 6, 7, 8, or 9 Cheerios.

Procedure

The procedure was identical to that in previous studies (vanMarle, 2013; vanMarle & Wynn, 2011). Infants sat on the floor in front of their parent, facing an experimenter approximately 6 ft. away. Infants first completed one warm-up trial in which they watched an experimenter hide a toy in a bucket and were encouraged to retrieve it. One test trial followed. First, the experimenter brought out two empty cups, briefly shook them upside down to show they were empty, and placed them on the floor simultaneously, right side up, about 3 ft. apart and about halfway between themselves and the infant. Next, the experimenter modeled eating for the infant by showing four Cheerios in their hand, and eating them one at a time while saying, “Yum, yum!” Finally, the experimenter hid Cheerios in each cup one at a time, first one amount, and then the other. Infants only ever saw one O at a time. Once both amounts were hidden, infants were encouraged to crawl/walk to the cup of their choice. The experimenter looked down into his/her lap during the response period to avoid cuing the infant. If infants failed to approach the cups immediately, the experimenter verbally encouraged the infant by saying, “Can you find something to eat?” Encouragement continued until the infant chose a cup, or 20 seconds elapsed, and was equally enthusiastic regardless of the infant’s path of progress.

If infants could represent and compare the quantities, they were expected to select the larger amount (Feigenson et al., 2002; vanMarle, 2013; vanMarle & Wynn, 2011). The dependent measure was the amount chosen (smaller or larger). The experimenter manually recorded the infant's choice, and testing sessions were videotaped. Infants were considered to have made a choice when they had approached a cup and either reached into it, or stayed near it for at least 8 seconds. If infants failed to make a choice within 20 seconds, the experiment was terminated. The data from twenty infants (~30% of the sample) were randomly selected and recoded from the video records by a trained observer blind to the infants' condition (viewing only the choice period, after all Cheerios had been hidden); reliability was 100%.

Results

Preliminary analyses indicated there were no differences between males and females in any of the conditions (two-tailed Fisher exact tests, all *p*'s > .05). In addition, infants did not show any side (left or right) or order (first or second) biases (two-tailed binomial tests, all *p*'s > .05), nor did these variables interact (all *p*'s > .05). The only exception was a significant preference for choosing the right side quantity in the 7:8 condition. Since the side of the larger quantity was counterbalanced, however, this bias could not have affected the overall response pattern.

As predicted, infants reliably selected the larger amount in both 2:3 ratio conditions (4v6 and 6v9) – 12 of 16 infants in each condition, *p*s < .05, 1-tailed binomial test. In contrast, performance did not differ from chance for more difficult ratios, 3:4 (6v8, 10/16 infants) and 7:8 (7v8, 10/16 infants) (both *p*s > .05). This ratio-dependent performance implicates the ANS and is consistent with previous findings in which infants reliably discriminate large numbers differing by a 2:3 ratio, but not a 4:5 ratio, by 9 months of age (Lipton & Spelke, 2003). See

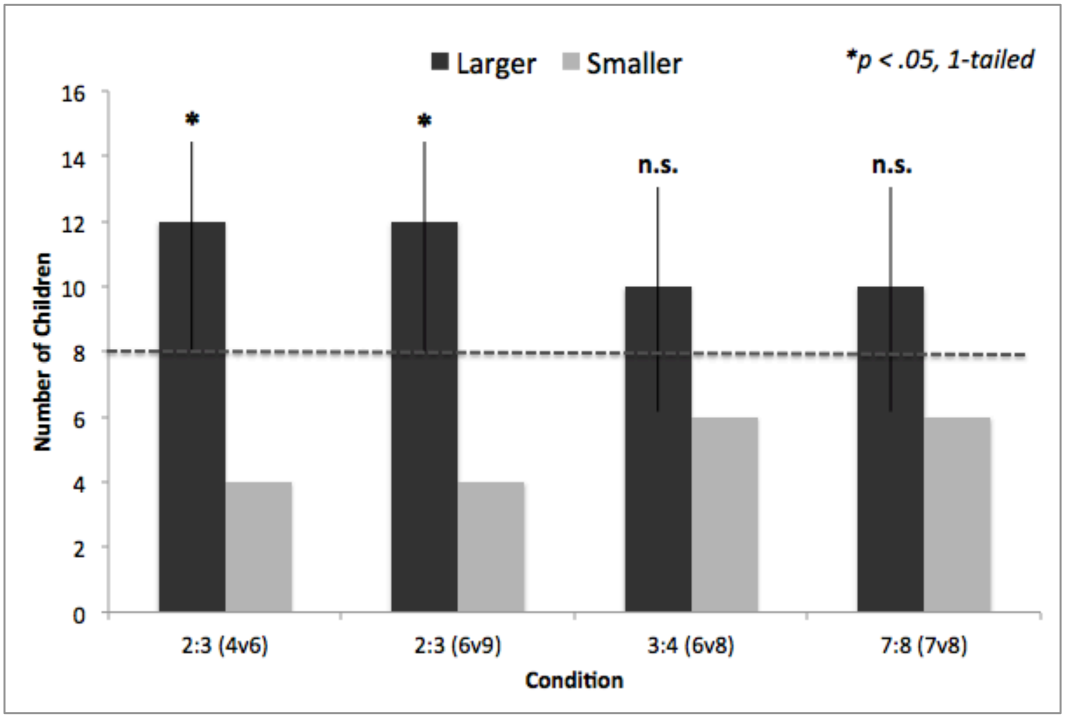

Figure 1. Number of infants in each condition choosing the larger and smaller amount. Infants reliably chose the larger amount when the quantities differed by a 2:3 ratio (4v6 and 6v9), but not when they differed by a 3:4 (6v8) or 7:8 ratio (7v8). Bars indicate 95% CIs.

Discussion

Our results show that infants' large number ordinal judgments are ratio-dependent, limited by the proportionate difference between the quantities. The discrimination function found for ordinal judgments here (success with 2:3, but not 3:4 or 7:8 ratios) mirrors that found for infants' numerical discrimination and duration discrimination at roughly this same age, suggesting that the ANS may underlie both discrimination and ordering of numerical quantities. Infant failure with 3:4 and 7:8 ratios cannot be explained as stemming from the greater overall number of items to be compared (and thus greater memory demands) since the 6v9 comparison had the same (or greater) total number of O's and took an equivalent amount of time to present. Thus, not only are infants able to use their ANS to discriminate between numbers of items (i.e., detect inequalities), they are also able to compare these magnitudes to make an ordinal judgment shown by their systematic selection of the larger quantity when the two amounts were discriminable. This is consistent with findings in adults in which cardinal and ordinal judgments over nonsymbolic quantities (dot arrays) was shown to activate the same areas in the brain (the interparietal sulcus; Lyons & Beilock, 2013). And it also consistent with previous findings in nonhuman primates using a similar task in which rhesus monkeys' ordinal judgments were shown to depend on ratio, implicating the ANS (Beran, 2007).

The ability to determine which of two food quantities is greater would have been highly adaptive in our species' evolutionary history. Many studies have demonstrated this ability in nonhuman primates (e.g., Beran, 2001; Beran, Johnson-Pynn, & Ready, 2011; Evans, Beran, Harris, & Rice, 2009), and crucially, nonhuman primates' performance is ratio dependent, even in the small number range (Beran, 2007; vanMarle, Aw, McCrink, & Santos, 2006). In contrast, findings with human infants suggest a discontinuity for small and large sets, with performance for small sets supported by the OTS (Feigenson et al., 2002; vanMarle, 2013), while performance with larger sets, as we show here, depends on the ANS. Given that the ANS is present across species and throughout development (Feigenson et al., 2004; Mou & vanMarle, 2014), the discontinuity seen in human infants begs for an explanation. How can adult humans and nonhuman primates (Cantlon & Brannon, 2006; Cordes, Gelman, Gallistel, & Whalen, 2001; Jones & Brannon, 2012) use the ANS to represent and order values across the entire number range , while human infants (and perhaps the young of some nonhuman species, e.g., Bisazza, Piffer, Serena, & Agrillo, 2010) utilize distinct mechanisms to make ordinal judgments for small and large sets (Mou & vanMarle, 2014; vanMarle, 2013)? One way we are trying to get at this question is by asking *when* in development infants become able to make cross-boundary judgments. Recent data from an ongoing study in our lab suggest that the ability may not develop until almost 2 years of age (vanMarle, Seok, & Mou, 2015). Knowing when infants develop this ability can provide insight into how the change occurs, and whether it is related to children's growing number knowledge.

It is an open question whether infants' judgments were based on number or continuous extent (including duration of presentation), which were confounded in our study. Previous studies suggest that number and volume/surface area compete for infants' attention when sets are small, with amount sometimes winning out when the dimensions are pitted directly against each other (Clearfield & Mix, 1999; 2001). However, because infants require a larger ratio difference to discriminate continuous quantity than number (Cordes & Brannon, 2008b), the discrimination threshold found here suggests the use of number, since continuous amount would likely have required a ratio difference more favorable than 2:3 for infants of this age. Similarly, previous studies that have controlled for duration of presentation have found that infants' performance does not rely on that cue (Feigenson & Carey, 2005). Nonetheless, we cannot strictly rule out either of these correlated cues.

In sum, the present study provides evidence that the ANS supports ordinal judgments in infancy by showing that these judgments are ratio-dependent in the large number range. The performance profile seen here matched that found in previous studies for numerical discrimination (Lipton & Spelke, 2003; Xu & Spelke, 2000), suggesting that the ANS supports not only cardinal knowledge (i.e., a sense of magnitude), but also ordinal knowledge, as is found for nonsymbolic quantities in adults (Lyons & Beilock, 2013). As such, the ordinal structure of the ANS and its use by infants for sets beyond the capacity limit of the OTS may set the stage for children to begin building an understanding of the ordinal nature of the verbal count list (Gallistel & Gelman, 1992, 2005; vanMarle et al., 2014).

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Footnote

[1] Five infants (3 male) contributed data to two of the four conditions, with the two data points separated by at least one week. Two of these infants were in the same presentation order on both occasions, but only one made the same choice (i.e., a male infant in the sm-L order chose the larger amount both times). Given that infants only had one test trial, it is not likely that infants' choices were based on a learned response.

Figure 1.

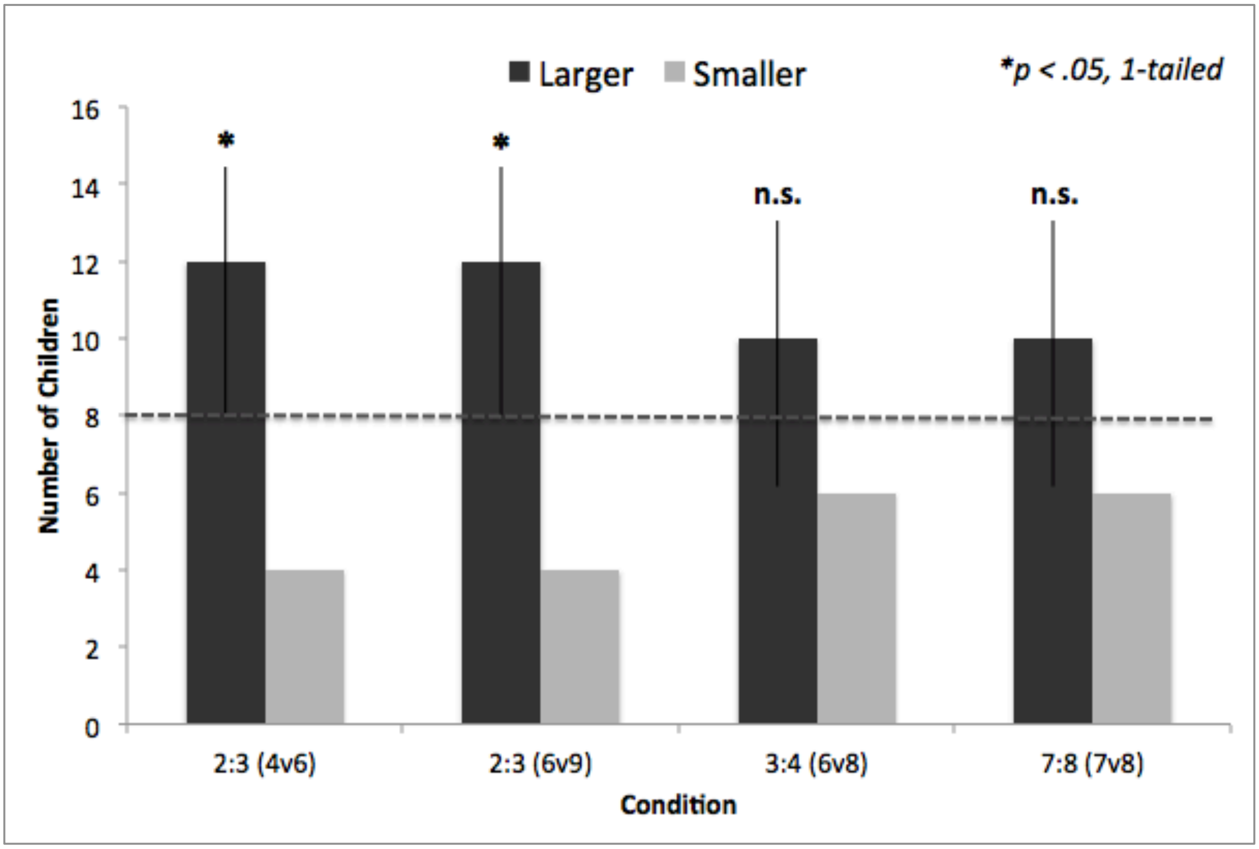

Figure 1. Number of infants in each condition choosing the larger and smaller amount. Infants reliably chose the larger amount when the quantities differed by a 2:3 ratio (4v6 and 6v9), but not when they differed by a 3:4 (6v8) or 7:8 ratio (7v8). Bars indicate 95% CIs.